A Generative Perspective on MRFs in Low-Level Vision

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Introduction

Goal:
- Understand generative properties of MRF models of natural images.

Questions:
- How can we evaluate generative MRF models?
- How well do typical image priors capture the statistical properties of natural images?

Previous methods:
- Use application-specific evaluation in the context of MAP estimation.
- Analyzing the statistical properties via single-site Gibbs sampling not very practical [9].

Our approach:
- Explore rapidly mixing Gibbs sampler for MRF priors [2, 3].
- Analyze the generative properties with efficient sampler.
- Learn better generative MRFs based on flexible GSM potentials.
- Use sampling-based MMSE estimation for inference.

Flexible MRF framework

Fields-of-Experts image prior [5]:

\[ p(x; \Theta) = \frac{1}{Z} e^{-E(x; \Theta)} \prod_{i=1}^{N} \Phi(x_i; \alpha_{ij}) \]

- Use flexible Gaussian scale mixtures (GSMs) as experts [8]:
  \[ \Phi(x; \alpha_{ij}) = \sum_{j=1}^{J} \alpha_{ij} \mathcal{N}(x; 0, \sigma_{ij}^2) \]
- Subsumes many popular pairwise and high-order models.

Efficient auxiliary-variable Gibbs sampler [2]:
- Retain the scales of the GSM as a hidden random vector \( \alpha \).
- Define a joint distribution \( p(x; \alpha; \Theta) \) such that:
  \[ \sum_{\alpha} p(x; \alpha; \Theta) = p(x; \Theta) \]
- This allows to define an auxiliary-variable Gibbs sampler that alternates between sampling \( \alpha \) and \( x \).

Some additional generative properties of our models:
- Random filter statistics and multi-scale derivative statistics.
- High-order model captures long-range dependencies.

Image restoration

Common method – MAP:

\[ x = \arg \max_{x} p(y; x; \Theta) = \arg \max_{x} p(x; \Theta) \]

- Regularization weight \( \lambda \) often used to improve performance.
- Point estimate - makes no use of uncertainty.
- Often see staircasing artifacts with heavy-tailed potentials.
- Low correlation between generative quality of the model and its performance in terms of image restoration.

Our method – MMSE:
- Bayesian maximum mean squared error estimate:
  \[ k = \arg \min_{k} \mathbb{E}[|x - k|^2 | y; \Theta] = \mathbb{E}[x | y; \Theta] \]
- Compute the average of a large number of posterior samples.
- The auxiliary-variable Gibbs sampler extends to the posterior for Gaussian likelihood models [3].
- Use multiple Markov chains to assess sampler convergence.
- Readily applies to image denoising, inpainting, ... etc.

Benefits from MMSE estimation:
- Improved quantitative results: PSNR, SSIM.
- No staircasing artifacts (piecewise-constant regions).
- Marginal statistics of output images match those of originals well.

Image denoising

- Assume additive white Gaussian noise with known variance.
- Sampling conditional distribution of scales \( k \) is straightforward.
- Conditional distribution of image \( x \) is a Gaussian
  \[ p(x; k; \Theta) = \mathcal{N}(x; \mu_k, \Sigma_k) \]
- For assessing convergence, run 4 parallel Markov chains.

Table 1. Denoising results [JPG/FNR=60] at 60 PSNR (dB) and SSIM (95% confidence intervals)

Summary

- Evaluated MRFs through their generative properties, based on a flexible framework with an efficient sampler.
- Common image priors are surprisingly poor generative models.
- Learned better generative MRFs (pairwise & high-order) - peakier potentials than considered before.
- Sampling-based MMSE estimation allows generic generative models to compete with recent discriminative models; the MMSE estimate even has a number of additional benefits.

References