A Generative Perspective on MRFs in Low-Level Vision

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Low-Level Vision

Super-Resolution

Discriminative
- ✔ performance
- ✗ versatility

Generative
- ✔ versatility
- ✗ learning

Image Restoration

Stereo

Optical Flow

Super-Resolution

Discriminative
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Low-Level Vision

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Generative MRF Priors

Stereo

Optical Flow
Common MRF Evaluation

- MRF prior $p(x)$
- Application likelihood $p(y|x)$
- Posterior $p(x|y)$

Indirect model evaluation

MAP estimation
(Gradient methods, Graph cuts, ...)

$\hat{x}$

Compare

$\rightarrow$ Measure PSNR, ...

Ground truth

$y$

generate/measure

MAP estimate
Desirable MRF Evaluation

- Purpose of MRF priors
  - Model statistical properties of natural images and scenes
- Evaluate generative properties [Zhu & Mumford '97]
  - e.g. derivative statistics of the model
  - neglected ever since

MRF prior

Draw samples (MCMC)

Data

Compare statistical properties

MRF samples

Difficult!
Agenda

1. Evaluate generative properties of common image priors
   • Pairwise & high-order MRFs
   • Based on a flexible MRF framework with an efficient sampler

2. Learn improved generative models

3. Find that in the context of MAP estimation our models do not perform as well as expected for image denoising

4. Address this problem (and others) by changing the estimator
Flexible MRF Model

- Fields-of-Experts (FoE) framework [Roth & Black ’05, ’09]
  - Subsumes popular pairwise & high-order MRFs

\[
p(x; \Theta) = \frac{1}{Z(\Theta)} e^{-\epsilon \|x\|^2 / 2} \prod_{c \in C} \prod_{i=1}^N \phi(J_i^T x(c); \alpha_i)
\]

- Image
- Telegram of experts [Roth & Black’05]
- Parameters
  \( \Theta = \{J_i, \alpha_i\}_{i=1}^N \)
- Expert function
- Linear filter
e.g. \[
\begin{pmatrix}
\text{Gray}
\end{pmatrix}
\]
- Vector of nodes in clique \( c \)
Flexible MRF Model

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\]

[Mixture Weights]

Gaussian Scale Mixture (GSM) Distribution

[Wainwright & Simoncelli ’99, Weiss & Freeman ’07]

\[
\phi(J_i^T x(c); \alpha_i) = \sum_{j=1}^J \alpha_{ij} \cdot N(J_i^T x(c); 0, \sigma_i^2 / s_j)
\]

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Sampling from the MRF

- Obtain joint distribution:
  - Product of GSMs = GSM
  - Augment MRF with auxiliary variables \( z \) for the mixture components and do not marginalize them out

- Gibbs sampling from the joint distribution \( p(x, z; \Theta) \) [Geman & Yang ’95; Welling et al. ’02]
  - Alternate block sampling from \( p(x|z; \Theta) \) and \( p(z|x; \Theta) \)
  - The \( z \) can be discarded in the end
  - Least-squares method for sampling \( p(x|z; \Theta) \) [Weiss ’05, Levi ’09]
MRF Sampling – Example

Pairwise MRF

High-order MRF with 3×3 cliques

Subsequent iterations of the Gibbs sampler
Generative Properties of Pairwise MRFs

- Consider simplest pairwise MRFs

Potential function

Derivative marginals

Fit to the marginals
Generalized Laplacian

Natural images

KLD=1.57
KLD=1.37

Marginal KL-divergence
Generative Properties of High-order MRFs

- Common FoE models
  - Evaluate filter statistics of model filters \( J_i \)

- Apparent contradiction:
  - Poor generative properties
  - Good application performance

Why?

[Roth & Black ’09] 24 5×5 filters
Student-t experts

[Weiss & Freeman ’07] 25 15×15 filters
fixed GSM experts

KLD=2.10
KLD=5.26

Natural/Images
MRF/Samples

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Learning Better Generative MRFs

- Learn shapes of flexible GSM experts and linear filters $J_i$ (for high-order model)
  - Use efficient sampler
  - Otherwise training similar to [Roth & Black ’09]

- Learned models:
  1. Pairwise MRF with single GSM potential (fixed first-derivative filters)
  2. FoE with $3 \times 3$ cliques and 8 GSM experts (including filters)
Generative Properties of Our Pairwise MRF

- Our pairwise MRF compared to previously shown

Potential function

Our learned GSM

Derivative marginals

Natural images KLD=0.006
Our Learned FoE in Comparison

Our learned $3 \times 3$ FoE

[Roth & Black ’09]

[Weiss & Freeman ’07]

Learned linear filters

Much more peaked!
Generative Properties of our FoE

- Filter statistics of our learned 3×3 FoE
  - Much better than previous models
  - Room for improvement
Image Denoising

- Image denoising assuming i.i.d. Gaussian noise with known standard deviation $\sigma$

$$p(x|y; \Theta) \propto \mathcal{N}(y; x, \sigma^2 I) \cdot p(x; \Theta)$$

Regularization weight

$[\text{Roth & Black '09}]
\text{MAP, optimal } \lambda$

PSNR\(=22.18\)\,dB
\quad PSNR\(=26.64\)\,dB
\quad PSNR\(=29.18\)\,dB
\quad PSNR\(=30.06\)\,dB
Recent works point to deficiencies of MAP [Nikolova ’07, Woodford et al. ’09]

We find only modest correlation between:

- Image quality of the MAP estimate
- Generative quality of the MRF

Better generative properties $\not\iff$ Better application performance
Image Denoising – MMSE

- We propose to use Bayesian minimum mean squared error estimation (MMSE)
  \[ \hat{x} = \arg \min_{\tilde{x}} \int \|\tilde{x} - x\|^2 p(x|y; \Theta) \, dx = E[x|y] \]

- [Levi ’09] extended sampler to the posterior
  - Only used a single sample in applications

- We approximate the MMSE estimate
  - Average samples from the posterior

- We find high correlation between:
  - Image quality of the MMSE estimate
  - Generative quality of the MRF
**Image Denoising – Results**

- Compared the MMSE estimate for our learned models with other popular methods

  Average PSNR (dB) for 68 test images ($\sigma = 25$)

<table>
<thead>
<tr>
<th>Method</th>
<th>PSNR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5×5 FoE [Roth &amp; Black ’09] – MAP w/$\lambda$</td>
<td>27.44</td>
</tr>
<tr>
<td>Non-local means [Buades et al. ’05]</td>
<td>27.50</td>
</tr>
<tr>
<td>pairwise (ours) – MMSE</td>
<td>27.54</td>
</tr>
<tr>
<td>5×5 FoE [Samuel &amp; Tappen ’09] – MAP</td>
<td>27.86</td>
</tr>
<tr>
<td>3×3 FoE (ours) – MMSE</td>
<td>27.95</td>
</tr>
<tr>
<td>BLS-GSM [Portilla et al. ’03] – (MMSE)</td>
<td>28.02</td>
</tr>
</tbody>
</table>

26.5 27.0 27.5 28.0
Advantages of the MMSE

- Denoising performance highly correlated with the generative quality of the model
- No regularization weight $\lambda$ required to perform well
- Denoised image does not exhibit incorrect statistics
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- Denoised image does not exhibit incorrect statistics
  - No piecewise constant regions

![Derivative marginals](chart.png)

Original images (noise-free)
Advantages of the MMSE

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- Denoised image does not exhibit incorrect statistics
  - No piecewise constant regions

[Woodford et al. ’09]
Advantages of the MMSE

- Denoising performance highly correlated with the generative quality of the model
- No regularization weight $\lambda$ required to perform well
- Denoised image does not exhibit incorrect statistics
  - No piecewise constant regions
  - Works with standard MRFs
Summary

- Evaluated MRFs through their generative properties
  - Based on a flexible framework with an efficient sampler
- Common image priors are surprisingly poor generative models
- Learned better generative MRFs (pairwise & high-order)
  - Potentials more peaked
- Sampling makes MMSE estimation practical
  - Several advantages over MAP
  - Excellent results from generative, application-neutral models
Thanks!

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Please come to our poster!


Questions?
More Generative Properties

Random filters

Multiscale derivative filters

Natural images

Our pairwise MRF

Our 3×3 FoE

KLD